

## DOMINANT STRATEGY MECHANISMS AND INCOMPLETE INFORMATION

JOHN LEDYARD\*

### 1. Introduction

Historically much intellectual effort has been devoted to discover whether or not it is possible to design an ideal informationally decentralized mechanism to aggregate individual values into some optimal group decision. While the terms 'ideal' and 'optimal' have been defined in many ways, recently the search has been narrowed to nondictatorial, nonimposed, dominant strategy mechanisms. The basis for the first two properties is obvious. The basis for the latter property is the desire, or necessity, to protect the group from individuals' incentives to manipulate (usually through preference misrepresentation) the mechanism and, thus, produce nonoptimal decisions. We now know that it is impossible to find such an ideal mechanism. This impossibility has been formalized in the work of Gibbard [5] and Satterthwaite [13] in the context of social choice theory, in the work of Hurwicz [9] in the context of resource allocation with private goods, and in the work of Green and Laffont [6], Hurwicz [10], Roberts [12] and Walker [15] in the context of resource allocation with public goods.

In apparent contradiction to the above results, d'Aspremont and Gérard-Varet [2, 3] and Arrow [1] have discovered a mechanism which is a nonimposed, nondictatorial, dominant strategy mechanism if agents are incompletely informed about the characteristics of other agents in the economy. Intuitively one might conjecture that the role of incomplete information is to make risk-averse agents reluctant to attempt to manipulate some mechanisms, since they can no longer be sure of achieving individual gains, *ex post*. In this paper we explore this conjecture in some detail and find it to be incorrect. In particular, we ask whether a mechanism which is not a dominant strategy mechanism under complete information can become a dominant strategy mechanism under incomplete information.

\*Both the National Science Foundation (Grant SOC-76-20953) and the Fairchild Foundation are gratefully acknowledged for their support.

In section 2 we introduce the formal models of group decision mechanisms under complete and incomplete information. In section 3 the results of our inquiry are presented in a sequence of theorems. The main conclusions are: (1) a mechanism is a dominant strategy mechanism under complete information if and only if it is a dominant strategy mechanism under incomplete information when a sufficiently rich range of prior beliefs are to be covered, and (2) for continuously differentiable mechanisms in environments with continuously differentiable utility functions, a mechanism is not a dominant strategy mechanism under complete information if and only if it is not a dominant strategy mechanism under incomplete information even if the range of prior beliefs to be covered is any nonempty open subset of consistent beliefs which are representable by positive, continuous density functions. Thus, only if prior beliefs are restricted to a closed, nowhere dense subset of conceivable priors can one convert a mechanism which lacks the dominant strategy property under complete information to one which has the dominant strategy property under complete information.

In section 4 we conclude with some remarks concerning the apparent incompatibility of these results and those of d'Aspremont and Gérard-Varet [2, 3] and Arrow [1]. In particular, the fact which explains the difference is that they allow as inputs to the mechanism not only messages from each agent but also the known independent probabilities of different possible utility functions for the agents, while we allow only messages sent by the agents. A final observation on the sensitivity of their results, with respect to the assumption that the probabilities used by the mechanism are precisely those held a priori by all agents, is also presented in this section.

## 2. Environments and mechanisms

A complete information economic environment is an  $N+2$ -tuple  $\langle I, X, e^1, \dots, e^n \rangle$ , where  $I$  is the index set of  $n$  agents,  $X$  is the feasible outcome space<sup>1</sup> and  $e^i$  is the characteristic of agent  $i$  (preferences, endowments, etc.). We let  $E = E^1 \times \dots \times E^n$  be the space of complete information environments (given  $I$  and  $X$ ) where  $E^i$  is the set of possible characteristics of the agent  $i$ .

An allocation mechanism is a 4-tuple  $\langle I, M, X, h \rangle$ , where  $M$  is the 'language' and  $h$  is a function from the Cartesian product  $M^1 \times \dots \times M^n$  to  $X$ .  $I$  and  $X$  are as before. In particular,  $h(m^1, \dots, m^n) \in X$  is the outcome if each agent communicates the message  $m^i$ . Of particular interest in this paper are dominant strategy mechanisms. To define this class of mechanisms precisely, we introduce the following assumption.

<sup>1</sup>In most economic models the set of feasible outcomes is a function of  $e = (e^1, \dots, e^n)$ , particularly through endowments. We ignore this difficulty here since the results below are not altered by doing so.

C.0: Given  $\langle I, X, E \rangle$ , where  $E^0 \equiv E^1 = \dots = E^n$ , there is a real-valued (utility) function  $u : X \times I \times E^0 \rightarrow \mathbb{R}$  such that  $u(x, i, e^0)$  is the  $i$ th agent's utility for  $x$  if  $e^i = e^0$ .

**Definition 1.** Given  $\langle I, X, E \rangle$ , a mechanism  $\langle I, M, X, h \rangle$  is said to be a dominant strategy mechanism on  $E$  if and only if there exists a function for each  $i$ ,  $\delta^i : E^i \rightarrow M^i$ , such that for all  $m \in M^1 \times \dots \times M^n$ ,  $e^i \in E^i$ ,

$$u[h(m/\delta^i(e^i)), i, e^i] \geq u[h(m), i, e^i],$$

where  $(m/\delta^i(e^i)) = (m^1, \dots, m^{i-1}, \delta^i(e^i), m^{i+1}, \dots, m^n)$ .

A trivial dominant strategy mechanism is  $h(m) = x_0 \in X \forall m \in M$ . Another is,  $h(m) = f(m^{i_0})$  for some  $i_0 \in I$ , for all  $m \in M$ . The first is an 'imposed' solution, the second is dictatorial. According to the Gibbard-Satterthwaite theorem [5, 13] these are the only types of dominant strategy mechanisms, if  $E^0$  contains enough variation in preferences. In this paper we consider whether the introduction of incomplete information leads to a larger class of dominant strategy mechanisms. My feeling is that this is most likely to be true if agents have to choose their messages without recourse.

The simplest way to model the problem of an agent who lacks complete information and who must choose his message  $m^i \in M^i$  without recourse is to assume  $i$  has some subjective beliefs about the parameters he does not know and cannot control.

To do so, we must broaden the interpretation of an agent's characteristic to include not only preferences but also beliefs.

Let  $S^i = M^1 \times \dots \times M^{i-1} \times M^{i+1} \times \dots \times M^n$ . We will maintain the following assumption throughout.

I.0: For each  $i = 1, \dots, n$ ,

(a)  $S^i$  is endowed with a Hausdorff topology.  $\mathcal{M}(S^i)$  is the space of all probability measures on  $S^i$ .  $\mathcal{M}(S^i)$  is endowed with the topology of weak convergence.  $X$  and  $\mathcal{M}(X)$  are similarly topologized.

(b) There is a bounded, measurable (Von Neumann-Morgenstern) utility function,  $u : X \times I \times E^0$  such that if  $\mu, \hat{\mu} \in \mathcal{M}(X)$  are two gambles on  $X$ , then  $i$  prefers  $\mu$  to  $\hat{\mu}$  when  $e^i = e^0$  if and only if

$$\int u[x, i, e^i] d\mu > \int u(x, i, e^i) d\hat{\mu}.$$

(c) There is a function  $\psi^i : E^i \rightarrow \mathcal{M}(S^i)$  such that  $\psi^i(e^i)$  represents  $i$ 's beliefs about others' messages given  $e^i$ .

(d) We consider only allocation mechanisms such that  $h$  is a Borel-measurable function.

An incomplete information environment is the  $2(n+1)$ -tuple

$$\langle I, X, e^1, \dots, e^n, \psi^1, \dots, \psi^n \rangle$$

and  $I \times X \times E \times \Psi \equiv I \times X \times E^1 \times \cdots \times E^n \times \Psi^1 \times \cdots \times \Psi^n$  is a class of incomplete information environments.<sup>2</sup>

**Definition 2.** Given  $\langle I, X, E, \Psi \rangle$ , a mechanism  $\langle I, M, X, h \rangle$  is said to be a dominant strategy mechanism on  $E \times \Psi$  if and only if there exists a function for each  $i$ ,  $\delta^i: E^i \rightarrow M^i$ , such that for all  $m^i \in M^i$  and all  $(e^i, \psi^i) \in E^i \times \Psi^i$ ,

$$\int u[h(m/\delta^i(e^i)), i, e^i] d\psi^i(e^i) \geq \int u[h(m), i, e^i] d\psi^i(e^i).$$

We might have allowed  $\delta^i$  to depend on  $\psi^i$  as well as on  $e^i$  since  $\psi^i$  is part of the (private) description of the agent  $i$ . However, if we did so then all mechanisms would be dominant strategy mechanisms under that definition. Furthermore, the main interest in dominant strategy mechanisms is the hope that some specific performance can be guaranteed. For example, one might wish to find a mechanism such that  $h[\delta^1(e^1), \dots, \delta^n(e^n)]$  is Pareto-optimal in  $e = (e^1, \dots, e^n)$  for all  $e \in E$  where  $\delta^i$  is the appropriate dominant strategy. If so, then  $\delta^i$  certainly must be constant on the various possible expectations functions,  $\psi^i$ . Thus, we restrict the domain of  $\delta^i$  to  $E^i$ .

We now turn to the relationship between the class of dominant strategy mechanisms on  $E$  and the class of those on  $E \times \Psi$ .

### 3. Equivalence

Our first result is not very surprising but is presented for completeness.

**Theorem 1.** Given  $\langle I, X, E, \Psi \rangle$  where  $\Psi^i$  is all functions  $\psi^i: E^i \rightarrow \mathcal{M}(S^i)$ , a mechanism  $\langle I, M, X, h \rangle$  is a dominant strategy mechanism on  $E \times \Psi$  if and only if  $\langle I, M, X, h \rangle$  is a dominant strategy mechanism on  $E$ .

*Proof.* (if) This is obvious since if

$$u[h(m/\delta^i(e^i)), i, e^i] \geq u[h(m), i, e^i], \quad \text{for all } m,$$

then

$$\int u[h(m/\delta^i(e^i)), i, e^i] d\psi^i(e^i) \geq \int u[h(m), i, e^i] d\psi^i(e^i),$$

for any  $\psi^i \in \Psi^i$ .

(only if) Suppose  $\langle I, M, X, h \rangle$  is not a dominant strategy mechanism on  $E$ . Then  $\exists m, \hat{m} \in M$  and  $e^i \in E^i$  such that

$$u[h(\hat{m}), i, e^i] > u[h(\hat{m}/m^i), i, e^i]$$

<sup>2</sup>This model of incomplete information is different from that of Harsanyi [8]. Later in this section we introduce a concept of consistency of expectations which gives the Harsanyi model as a special case of the present one.

and

$$u[h(m), i, e^i] > u[h(m/\hat{m}^i), i, e^i].$$

Let  $\psi^i$  be any function such that  $\psi^i(e^i)(A) = 1$  if  $s^i \in A$  and let  $\hat{\psi}^i$  be any function such that  $\hat{\psi}^i(e^i)(A) = 1$  if  $\hat{s}^i \in A$ . Then

$$\int u[h(\hat{m}), i, e^i] d\hat{\psi}^i(e^i) > \int u[h(\hat{m}/\hat{m}^i), i, e^i] d\hat{\psi}^i(e^i)$$

and

$$\int u[h(m), i, e^i] d\psi^i(e^i) > \int u[h(m/\hat{m}^i), i, e^i] d\psi^i(e^i).$$

Thus,  $(I, M, X, h)$  is not a dominant strategy mechanism on  $E \times \Psi$ . Q.E.D.

**Remark.** The if statement is valid no matter what subset of  $\Psi$  is employed. The only if statement seems to rely heavily on the existence in  $\Psi$  of functions which map into measures concentrated on single points. As we will see later, however, this reliance is only illusory. For now, however, the triviality of the theorem can be seen by translating the proof of sufficiency into words: if an agent's optimal response when others send  $s^i$  differs from that when others send  $\hat{s}^i$  then his response if he believes  $s^i$  will occur with probability 1 will differ from that if he believes  $\hat{s}^i$  will occur with probability 1.

**Remark.** The crucial conclusion of Theorem 1 is that a mechanism will be a dominant strategy mechanism under incomplete information only if it is a dominant strategy mechanism under complete information. The crucial hypothesis is that  $\Psi^i$  contain enough functions which map into single point distributions. Two obvious questions remain. First, what happens if  $\Psi^i$  is restricted to functions mapping into diffuse priors?<sup>3</sup> This is important since it is certainly in the spirit of incomplete information to require agents to believe that all events are possible. Secondly, what happens if  $\psi^i$  is required to be consistent, in some fashion, with the model? In both cases it might be that restrictions on  $\Psi^i$  will cause the conclusion of Theorem 1 to be false.

To answer the question about diffuse priors, we consider the smaller set of priors which can be represented by continuous positive density functions. We let  $\mathcal{M}^+(S^i)$  be the set of probability measures such that  $\eta \in \mathcal{M}^+(S^i)$  if and only if there is a continuous function  $g^i: S^i \rightarrow \mathbf{R}$  such that  $g^i(s^i) > 0$  for all  $s^i \in S^i$  and  $\eta(A) = \int_A g^i(s^i) ds^i$  for all Borel subsets  $A \subseteq S^i$ . We let  $\Psi^{i+}$  be the set of all functions whose range is  $\mathcal{M}^+(S^i)$ .

For ease of exposition we assume throughout the rest of the paper,

A.1:  $X \subseteq \mathbf{R}^K$ ,  $M^i \subseteq \mathbf{R}^L$ , for  $i = 1, \dots, n$ ,  $0 < K, L < \infty$ , where, furthermore,  $X$  and  $M^i$  are nonempty open subsets.

<sup>3</sup>A measure  $\eta \in \mathcal{M}(S^i)$  is diffuse if  $\eta(\{s^i\}) = 0 \forall s^i \in S^i$ .

We can now state and prove the following theorem, indicating that requiring diffuse priors does not in any way blunt the conclusions of Theorem 1.

**Theorem 2.** Given  $\langle I, E, X, \Psi \rangle$  such that  $\Psi \equiv \langle \Psi^{1+}, \dots, \Psi^{n+} \rangle$  and  $u(x, i, e^0)$  is continuous in  $x$ , and given a mechanism  $\langle I, M, X, h \rangle \equiv R$  such that  $h$  is continuous in  $m = (m^1, \dots, m^n)$  then  $R$  is a dominant strategy mechanism on  $E \times \Psi$  if and only if  $R$  is a dominant strategy mechanism on  $E$ .

*Proof.* (if) Proven as in Theorem 1.

(only if) Suppose  $R$  is not a dominant strategy mechanism on  $E$ .

$$\exists m, \hat{m}, e^i, i \ni u[h(m), i, e^i] > u[h(m/\hat{m}^i), i, e^i]$$

and

$$[h(\hat{m}), i, e^i] > u[h(\hat{m}/m^i), i, e^i].$$

Let  $f_\sigma: S^i \rightarrow \mathbf{R}$  be the density of the multivariate normal<sup>4</sup> with mean  $(\hat{m}^1, \dots, \hat{m}^{i-1}, \hat{m}^{i+1}, \dots, \hat{m}^n)$  and variance-covariance matrix  $\sigma I$ . By continuity of  $u$  in  $x$  and  $h$  in  $m$ , if  $\sigma$  is small enough, but positive, then

$$\int u[h(m), i, e^i] f_\sigma(s^i) ds^i > \int u[h(m/\hat{m}^i), i, e^i] f_\sigma ds^i$$

and

$$\int u[h(\hat{m}), i, e^i] \hat{f}_\sigma(s^i) ds^i > \int u[h(\hat{m}/m^i), i, e^i] \hat{f}_\sigma ds^i.$$

Q.E.D.

One might still feel that Theorem 2 allows too many priors in that  $f_\sigma$  and  $\hat{f}_\sigma$  are significantly different and, of course, optimal strategies will vary. However, the next result indicates that one only need consider an open subset of  $\Psi^+$ , no matter how small, if one is willing to narrow slightly the class of environments and mechanisms.

**Theorem 3.** Given  $\langle I, E, X, \Psi \rangle$ , where  $\Psi$  is a nonempty open subset of  $\Psi^+$  and  $u(x, i, e^0)$  has continuous first derivatives in  $x$  for all  $(x, i, e^0)$  and given  $R = \langle I, M, X, h \rangle$  such that  $h$  has continuous first derivatives in  $m$  and  $u[h(m/\hat{m}^i), i, e^i] \geq u[h(m/\bar{m}^i), i, e^i]$  for all  $\bar{m}^i \in M^i$  if and only if<sup>5</sup>  $\nabla_m u[h(m/\hat{m}^i), i, e^i] = 0$ , then  $R$  is a dominant strategy mechanism in  $E \times \Psi$  if and only if  $R$  is a dominant strategy mechanism on  $E$ .

*Proof.* (if) Proven as in Theorem 1.

(only if) Suppose  $R$  is not a dominant strategy mechanism on  $E$ . Then there are:  $i$  and  $e^i$  such that for all  $m^i \in M^i$  there is  $\hat{m}$  such

<sup>4</sup>If  $S^i$  is a proper subset of  $\mathbf{R}^{(n-1)L}$ , then let  $f_\sigma$  be the density of the appropriate multi-variate normal conditioned on the set  $S^i$ . A similar adjustment should be made throughout. Since nothing is altered by this adjustment, we proceed as if  $S^i = \mathbf{R}^{(n-1)L}$ .

<sup>5</sup> $\nabla_m f(m/\hat{m}^i) = [\partial f(m)/\partial m^1, \dots, \partial f/\partial m^i]$ .

that  $\nabla_m u[h(\hat{m}/m^i), i, e^i] \neq 0$ . Now suppose

$$\int \nabla_m u[h(m/\delta^i(e^i)), i, e^i] d\Psi^i(e^i) = 0, \text{ for some } \Psi^i \in \Psi^i.$$

If not, we are through. Let  $g$  be the continuous density representation of  $\Psi^i(e^i)$ , let  $\hat{m}$  be such that  $\nabla_m [h(\hat{m}/\delta^i(e^i)), i, e^i] \neq 0$  and let  $f_\sigma$  be the multivariate normal with mean  $\hat{m}$  and variance-covariance matrix  $\sigma I$  where  $\sigma$  is chosen small enough so that

$$\int \nabla_m u[h(m/\delta^i(e^i)), i, e^i] f_\sigma(s^i) ds^i \neq 0.$$

Now let  $G_\lambda(s^i) = \lambda g(s^i) + (1 - \lambda)f_\sigma(s^i)$  for all  $s^i$ . For  $\lambda$  near 1,  $G_\lambda(s^i)$  is the density function of  $\Psi^i(e^i)$  for some  $\Psi^i \in \Psi^i$ . Furthermore,

$$\int \nabla_m u[h(m/\delta^i(e^i)), i, e^i] G_\lambda(s^i) ds^i \neq 0, \text{ for all } \lambda \in (0, 1).$$

The desired conclusion follows.

Q.E.D.

**Remark.** What we have shown is that for differentiable mechanisms in differentiable environments, if the mechanism is not a dominant strategy mechanism under complete information in some  $e^i \in E^i$ , then under incomplete information small perturbations in priors lead to changes in the optimal strategy and thus the mechanism is not a dominant strategy mechanism under incomplete information in  $e^i \times \Psi^i$  where  $\Psi^i$  is any nonempty open subset of  $\Psi^{i+}$ .

The remaining issue concerns the impact of some type of consistency hypothesis on our results. At this point the work of Harsanyi [8] and others on incomplete information games becomes important. We will use the concept of Bayes equilibrium coupled with consistency across agents of their view of the situation to formalize our consistency requirement.

**Definition 3.** The vector of expectations functions  $\langle \psi^1, \dots, \psi^n \rangle$  is said to be consistent if and only if there exists a probability measure  $\pi \in \mathcal{M}(E^1 \times \dots \times E^n)$  and a vector of strategies  $\langle \delta^1, \dots, \delta^n \rangle$  where  $\delta^i: E^i \rightarrow M^i$  such that for  $i = 1, \dots, n$

(a)  $\delta^i(e^i)$  solves

$$\text{Max}_m \int u[h(\delta(e)/m^i), e^i] d\pi(\cdot | e^i) \text{ and}$$

(b)  $\psi^i(\hat{e}^i)(A) = \pi(\{e | \hat{e}^i = e^i, \delta^{j(i)}(e) \in A\})$  for all  $\hat{e}^i \in E^i$ ,  $A \subseteq S^i$ , where  $\delta(e) = [\delta^1(e^1), \dots, \delta^n(e^n)]$  and  $\delta^{j(i)}(e) = [\delta^1(e^1), \dots, \delta^{i-1}(e^{i-1}), \delta^{i+1}(e^{i+1}), \dots, \delta^n(e^n)]$ .

Essentially this means that all  $i$  have a consistent view of the economy,  $\pi$ ,

and use a Bayes equilibrium strategy,  $\delta$ . We let  $\Psi^c$  be the set of consistent expectations functions. Clearly,  $\Psi^c \neq \emptyset$  and  $\Psi^c \cap \Psi^+ \neq \emptyset$ .

Finally we need to introduce the concept of a weak dominant strategy mechanism.

**Definition 4.** Given  $\langle I, X, E \rangle$  a mechanism  $\langle I, M, X, h \rangle$  is said to be a weak dominant strategy mechanism if and only if for  $i = 1, \dots, n$  there is a function  $\delta^i: E^i \rightarrow M^i$  such that

$$u[h(m/\delta^i(e^i)), i, e^i] \geq u[h(m), i, e^i]$$

for all  $m^i \in M^i$  and for all  $m^{(i)} = (m^1, \dots, m^{i-1}, m^{i+1}, \dots, m^n)$  such that  $m^h \in \delta^h(E^h)$  for all  $h \neq i$ .

**Remark.** The only difference between this and Definition 1 is that only  $m^{(i)} \in \delta^{(i)}(E)$  are considered, whereas in Definition 1 all  $m^{(i)} \in M^{(i)}$  were. Since  $\delta(E)$  can be strictly contained in  $M$ , more mechanisms can be weak dominant strategy mechanisms than can be dominant strategy mechanisms.<sup>6</sup>

**Theorem 4.** Given an environment  $\langle I, X, E, \Psi \rangle$  which satisfies the assumptions of Theorem 3 and such that  $\Psi$  is a nonempty open subset of  $\Psi^c \cap \Psi^+$ , then  $R$  is a dominant strategy mechanism on  $E \times \Psi$  if and only if  $R$  is a weak dominant strategy mechanism on  $E$ .

*Proof.* (if) Proven as in Theorem 1 since if  $R$  is a weak dominant strategy mechanism and  $\delta^*$  is that dominant strategy and  $\psi \in \Psi^c$  then  $(\delta^*, \pi)$  is the appropriate pair upon which  $\psi$  is based. Thus, if

$$u[h(m/\delta^{*i}(e^i)), i, e^i] \geq u[h(m), i, e^i]$$

for all  $m^{(i)} \in \delta^{(i)}(E)$  and  $m^i \in M^i$ , then

$$\int u[h(\delta^*(e), i, e^i) d\pi(\cdot | e^i)] \geq \int u[h(\delta^*(e)/m^i), i, e^i] d\pi(\cdot | e^i)$$

for all  $m^i \in M^i$  and all  $\pi \in \mathcal{M}(E)$ .

(only if) If  $R$  is a dominant strategy mechanism on  $E \times \Psi$  where  $\Psi \subseteq \Psi^c \cap \Psi^+$ , and  $\delta^*$  is the dominant strategy, then

$$\int \nabla_m u[h(\delta^*(e)), i, e^i] d\pi(\cdot | e^i) = 0 \quad (1)$$

for all  $\pi$  such that  $(\delta^*, \pi)$  is the basis for some  $\psi \in \Psi$ . Now if  $R$  is not a weak dominant strategy mechanism on  $E$  then for some  $e, i$

$$\nabla_m u[h(\delta^*(e)), i, e^i] \neq 0.$$

<sup>6</sup>In Ledyard [11],  $M^i = E^i$  for all  $i$  and incentive compatibility required that  $\delta^i(e^i) = e^i$  be a dominant strategy. Thus,  $\delta(E) = E$  and the type of slippage which Definition 4 allows for was never an issue.



As in Theorem 3 we can perturb any  $\pi$ , such that (1) is true, to (say)  $\pi_\lambda$  such that

$$\int \nabla_m u[h(\delta^*(e)), i, e^i] d\pi_\lambda(\cdot | e^i) \neq 0. \quad (2)$$

Thus, even if (1) is true at  $e \times \psi$  we can find a  $\psi_\lambda$  in any open neighborhood of  $\psi$  (i.e.  $\psi_\lambda \in \Psi^c$ ) such that (2) is true. Therefore,  $R$  cannot be a dominant strategy mechanism on  $E \times \Psi$ . Q.E.D.

The main conclusion I have drawn from the preceding theorems is that the conjecture that the introduction of incomplete information would expand the class of dominant strategy mechanisms is false. Thus, if one likes continuous dominant strategy mechanisms, one seems to be confronted with the hard fact that, no matter what the informational structure, dictatorial or imposed outcome rules are, in general, the only candidates. The only time this may be false is if one searches the class of discontinuous outcome rules.

A second conclusion of the above analysis is that even if one is willing severely to restrict the space of possible prior beliefs to ensure that some mechanism is a dominant strategy mechanism under incomplete information when it is not under complete information, the appropriate restriction is indeed severe. It follows rather easily from the constructions in the proofs of Theorems 3 and 4 that for differentiable mechanisms  $R$  in differentiable environments  $E$ , if  $R$  is not incentive compatible on  $E$  but  $R$  is incentive compatible on  $E \times \Psi$ , where  $\Psi \subseteq \Psi^c \cap \Psi^+$ , then  $\Psi$  is a closed nowhere dense subset of  $\Psi^c \cap \Psi^+$ . I suspect there are few people who feel they can describe  $\psi$  with the precision necessary to ensure that  $\psi$  is in a closed nowhere dense subset. Thus, restrictions on priors do not, in general, produce dominant strategy mechanisms different from those that occur under complete information.

#### 4. Concluding remarks

An apparent contradiction of the conclusions in section 3 can be found in the work of d'Aspremont and Gérard-Varet [2, 3] and Arrow [1] who independently discovered a dominant strategy mechanism under incomplete information if there are no income effects. Their mechanism is a variant of the Clarke-Groves-Vickrey [4, 7, 14] mechanism which is not a dominant strategy mechanism under complete information if  $h(m)$  is required to be (weakly) feasible in the sense that resources are neither created nor destroyed; that is, all budgets balance. The reason the incomplete information mechanism is a dominant strategy mechanism in apparent contradiction to Theorem 4, is that  $h$  is allowed to depend on  $\pi$  as well as  $m$ . The theorems in this paper do not cover that type of mechanism.

In Ledyard [11] there is discussion of some of the philosophical issues

raised by the assumption that the probabilities of different possible utility functions are known and agreed on by all agents. Let me raise a more practical problem here. Suppose we take it on face value that there exists some  $\hat{\pi}$  representing the common prior beliefs. However, suppose that for some reason a different prior,  $\pi^*$ , is entered into the function  $h$  (small computational errors are sufficient to cause  $\pi^* \neq \hat{\pi}$ ). Letting  $h(m, \pi)$  be the d'Aspremont, Gérard-Varet, Arrow mechanism, they show that  $\delta^i(e^i) = e^i$  is a dominant strategy given beliefs  $\pi$ . That is to say

$$\int \nabla_m u[h(e, \pi), e^i] d\pi(\cdot | e^i) = 0,$$

for all  $e^i \in E^i$ .

We know from other work (e.g. Walker [15]) that there is some  $\hat{e} \in E$  such that

$$\nabla_m u[h(\hat{e}, \pi), \hat{e}^i] \neq 0$$

since otherwise  $h$  would be a feasible dominant strategy mechanism under complete information. Now using the techniques in the proof of Theorem 4 one can show, since  $h$  and  $u$  are continuously differentiable, that given any  $\pi^*$  and  $\pi$  such that

$$\int \nabla_m u[h(e, \pi^*), e^i] d\pi(\cdot | e^i) = 0$$

then in any nonempty open neighborhood of  $\pi$  there is another  $\pi'$  such that

$$\int \nabla_m u[h(e, \pi^*), e^i] d\pi'(\cdot | e^i) \neq 0.$$

It follows that for an open dense subset  $B \subseteq \mathcal{M}(E)$ , if  $\pi \in B$  then  $\delta^i(e^i) = e^i$  is not the optimal strategy for some  $i$ . Thus (generically) errors in discovering and computing the 'true' prior beliefs  $\pi$  lead to a loss of 'demand revelation' under incomplete information even if those errors are small.

It seems one must accept the inevitability of the generic impossibility of nondictatorial, nonimposed, dominant strategy mechanisms even under incomplete information. Accepting this, one is led quite naturally to ask second-best questions. It is quite possible that the d'Aspremont, Gérard-Varet, Arrow mechanism will do quite well in those inquiries; however, the answers will depend on future work.

## References

- [1] Arrow, K.J., 'The Property Rights Doctrine and Demand Revelation under Incomplete Information', Technical Report no. 243, Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford, CA., August 1977.
- [2] d'Aspremont, C. and L. Gérard-Varet, 'Individual Incentives and Collective Efficiency for an Externality Game with Incomplete Information', CORE D.P. 7519, Université Catholique de Louvain, 1975.

- [3] d'Aspremont, C. and L. Gérard-Varet, 'Incentives and Incomplete Information', CORE Discussion Paper 7705, Université Catholique de Louvain, 1977.
- [4] Clarke, E., 'Multi-part Pricing of Public Goods', *Public Choice*, 8 (1971) 19–33.
- [5] Gibbard, A., 'Manipulation of Voting Schemes: A General Result', *Econometrica*, 41 (1973) 587–601.
- [6] Green, J. and J.J. Laffont. 'Revelation des Preferences pour les Biens Publics', in: *Cahiers du Seminaire d'Econometrie* (C.N.R.S., Paris, 1977).
- [7] Groves, T., 'Incentives in Teams', *Econometrica*, 41 (1973) 617–631.
- [8] Harsanyi, J., 'Games with Incomplete Information Played by "Bayesian" Players, Parts I–III', *Management Science*, 14 (1967–1968) 158–182, 320–334, 486–502.
- [9] Hurwicz, L., 'On Informationally Decentralized Systems', in: R. Radner and C.B. McQuire, eds., *Decision and Organization* (North-Holland, Amsterdam, 1972).
- [10] Hurwicz, L., 'On the Existence of Allocation Systems Whose Manipulative Nash Equilibria are Pareto-optimal', Unpublished paper presented at the Third World Congress of the Econometric Society, Toronto, August 1975.
- [11] Ledyard, J., 'Incentive Compatibility and Incomplete Information', Social Science Working Paper no. 187, California Institute of Technology, October 1977.
- [12] Roberts, J., 'Incentives and Planning Procedures for the Provision of Public Goods', *Review of Economic Studies*, 44 (1977) forthcoming.
- [13] Satterthwaite, M., 'Strategy Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions', *Journal of Economic Theory*, 10 (1975) 187–217.
- [14] Vickrey, W., 'Counterspeculation, Auctions, and Competitive Sealed Tenders', *Journal of Finance*, 16 (1961) 8–37.
- [15] Walker, M., 'On the Impossibility of a Dominant-Strategy Mechanism to Optimally Decide Public Questions', Working Paper no. 160, Department of Economics, State University of New York at Stony Brook, 1976; to appear in *Econometrica*.